Modeling and simulation of contact between rigid bodies

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Navier-Stokes equations :

$$\left\{\begin{array}{rcl} \rho_f \frac{\partial u}{\partial t} + \rho_f (u \cdot \nabla) u - \nabla \cdot \sigma & = & 0_{\mathbb{R}^d}, \Omega, \\ \nabla \cdot u & = & 0, \quad \Omega, \end{array}\right.$$

where σ the total stress tensor :

$$\sigma = -pI_d + \mu[\nabla u + (\nabla u)^T].$$

Fluid definitions :

- $d = 2$ or $d = 3$, dimension.
- $\Omega \subseteq \mathbb{R}^d$, fluid domain.
- $u : \Omega \to \mathbb{R}^d$, velocity.
- $p : \Omega \to \mathbb{R}$, pressure.
- $\mu \in \mathbb{R}^+$, viscosity.
- $\rho_f \in \mathbb{R}^+$, density.
- $I_d \in \mathbb{M}_{d,d}$, identity tensor.

Fluid model with moving rigid bodies

The bodies can translate and rotate with gravity, fluid forces, contact and lubrication forces in body-body or body-wall interactions.

Newton equations :

$$m_i \frac{\partial U_i}{\partial t} = (m_i - \int_{B_i} \rho_f) g + F_i + F_i^c + F_i^l,$$
$$\frac{\partial [R J_i R^T \omega_i]}{\partial t} = T_i + T_i^c + T_i^l,$$

***i*-th body definitions :**

- $B_i \subseteq \mathbb{R}^d$, body domain.
- $m_i \in \mathbb{R}$, mass.
- $J_i \in \mathbb{R}^+$ if $d = 2$, else $J_i \succ 0 \in \mathbb{M}_{d,d}$, moment of inertia.
- $R \in \mathbb{M}_{d,d}$, rotation matrix.
- $U_i \in \mathbb{R}^d$, translational velocity.
- $\omega_i \in \mathbb{R}$ if $d = 2$, else $\omega_i \in \mathbb{R}^d$, angular velocity.
- $x_i^{CM} \in \mathbb{R}^d$, center of mass.

where $g \in \mathbb{R}^d$ is the gravity vector, $F_i^c, F_i^l \in \mathbb{R}^d$, T_i^c, $T_i^l \in \mathbb{R}^d$ the contact, lubrication forces and torque, and $F_i \in \mathbb{R}^d$, $T_i \in \mathbb{R}^d$ the hydrodynamic forces and torque acting on the body :

$$F_i = -\int_{\partial B_i} \sigma \cdot \overrightarrow{n} \quad \text{and} \quad T_i = -\int_{\partial B_i} (\sigma \cdot \overrightarrow{n}) \times (x_i - x_i^{CM}).$$

Fluid model with moving rigid bodies

Coupling condition on the velocities on ∂B_i :

$$u = U_i + \omega_i \times (x - x_i^{CM}).$$

Fluid-body problem :

$$\begin{cases} \rho_f \frac{\partial u}{\partial t} + \rho_f (u \cdot \nabla) u - \nabla \cdot \sigma &= 0_{\mathbb{R}^d}, \quad \text{in } \Omega, \\ \nabla \cdot u &= 0, \quad \text{in } \Omega, \\ u &= U_i + \omega_i \times (x - x_i^{CM}), \quad \text{on } \partial B_i, \\ m_i \frac{\partial U_i}{\partial t} &= (m_i - \int_{B_i} \rho_f) g + F_i + F_i^c + F_i^l, \\ \frac{\partial [R J_i R^T \omega_i]}{\partial t} &= T_i + T_i^c + T_i^l. \end{cases}$$

Decheng Wan, Stefan Turek. *Direct Numerical Simulation of Particulate Flow via Multigrid FEM Techniques and the Fictitious Boundary Method*, 2004.

Contact and lubrication forces

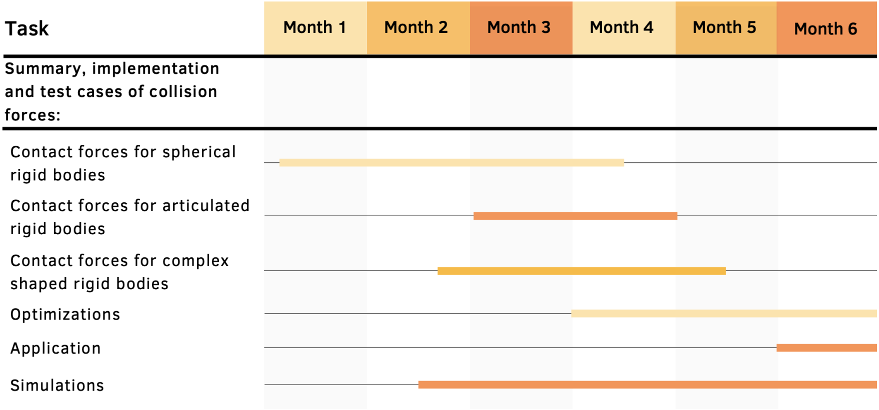
Contact forces

- Body surfaces are in direct contact.
- Approaches :
  - Hard sphere approach.
    [R. Jain, S. Tschisgale, J. Fröhlich, 2019]
  - Hertzian contact theory.
    [S. Ray, T. Kempe, J. Fröhlich, 2015]
  - Soft sphere approach.
    [M.N. Ardekani, P. Costa, W.P. Breugem, L. Brandt, 2016].

Lubrication forces

- Distance between body surfaces are very small.
- Ensure that hydrodynamic forces are resolved.
- Approaches :
  - Constant force.
    [R. Jain, S. Tschisgale, J. Fröhlich, 2019]
  - Repulsive force.
    [R. Glowinski, T.W Pan, T.I Hesla, D.D. Joseph, J. Périaux, 2000]

Gantt chart



Full Gantt chart : https ://feelpp.github.io/swimmer/swimmer/latest/StageCeline/Introduction.html

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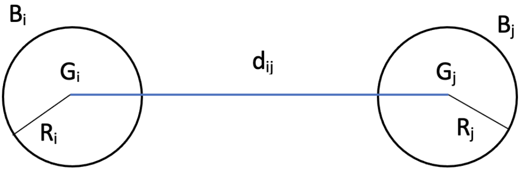
Smooth collision : The velocities of the rigid bodies coincide at the points of contact.



We note $d_{ij} = ||G_i - G_j||_2$, distance between the disk mass centers.

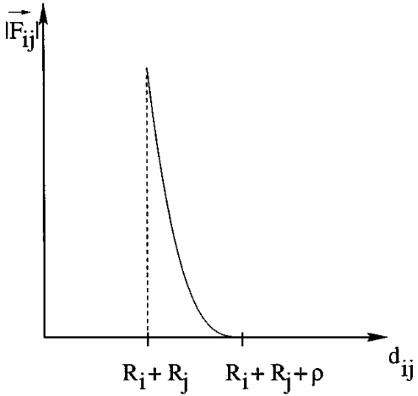
The repulsion force $\overrightarrow{F_{ij}}$ has to verify three properties :

1. $\overrightarrow{F_{ij}}$ is parallel to $\overrightarrow{G_i G_j}$.

R. Glowinski, T. W. Pan, T. I. Hesla, D. D. Joseph, and J. Périaux. *A Fictitious Domain Approach to the Direct Numerical Simulation of Incompressible Viscous Flow past Moving Rigid Bodies : Application to Particulate Flow*, 2000.

Collisions between spherical rigid bodies

2. $|\overrightarrow{F_{ij}}| = 0 \quad$ if $d_{ij} > R_i + R_j + \rho$, where ρ the range of the repulsion force.
3. For $R_i + R_j \leq d_{ij} \leq R_i + R_j + \rho$:



Repulsion force
[R. Glowinski, T.W Pan, T.I Hesla, D.D. Joseph, J. Périaux, 2000]

Collisions between spherical rigid bodies

Definition of $\overrightarrow{F_{ij}}$ for body-body interaction :

$$|\overrightarrow{F_{ij}}| = \begin{cases} 0\,, & \text{for} \quad d_{ij} > R_i + R_j + \rho, \\ \frac{1}{\epsilon}(G_i - G_j)(R_i + R_j + \rho - d_{ij})^2\,, & \text{for} \quad R_i + R_j \leq d_{ij} \leq R_i + R_j + \rho, \\ \frac{1}{\epsilon'}(G_i - G_j)(R_i + R_j - d_{ij})\,, & \text{for} \quad d_{ij} < R_i + R_j. \end{cases}.$$

where $(R_i + R_j + \rho - d_{ij})^2$ a quadratic activation term and $(G_i - G_j)$ gives the direction of the force.

The stiffness parameters are set to $\epsilon \approx h^2$ and $\epsilon' \approx h$, if :

- $\rho \approx h$, h the mesh step.
- $\frac{\rho_b}{\rho_f} \approx 1$, ρ_b the body density and ρ_f the fluid density.
- The fluid is sufficiently viscous.

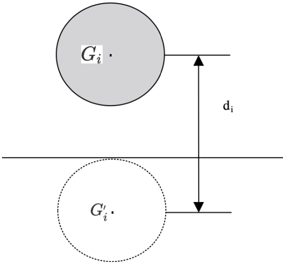
Decheng Wan, Stefan Turek. *Direct Numerical Simulation of Particulate Flow via Multigrid FEM Techniques and the Fictitious Boundary Method*, 2004.

Collisions between spherical rigid bodies

Definition of $\overrightarrow{F_i^W}$ for body-wall interaction :

$$|\overrightarrow{F_i^W}| = \begin{cases} 0\,, & \text{for} \quad d_i > 2R_i + \rho, \\ \frac{1}{\epsilon_W}(G_i - G_i^{'})(2R_i + \rho - d_i)^2\,, & \text{for} \quad 2R_i \leq d_i \leq 2R_i + \rho, \\ \frac{1}{\epsilon_W^{'}}(G_i - G_i^{'})(2R_i - d_i)\,, & \text{for} \quad d_i < 2R_i. \end{cases}.$$

where $\rho \approx h$, $\epsilon_W = \frac{\epsilon}{2}$ and $\epsilon_W^{'} = \frac{\epsilon^{'}}{2}$.



Imaginary body
[P. Singh, T.I. Hesla, D.D. Joseph, 2002]

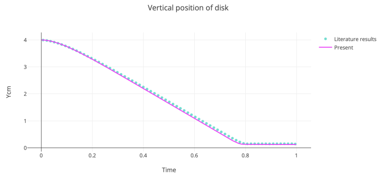
Total repulsion force applied on body B_i :

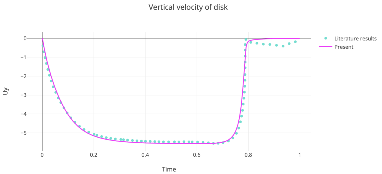
$$|\overrightarrow{F_i}| = \sum_{j=1, j\neq i}^{N} |\overrightarrow{F_{ij}}| + |\overrightarrow{F_i^W}|,$$

where N the number of bodies.

Simulation : Spherical body-bottom interaction

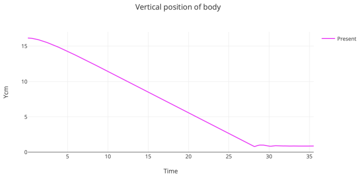
Simulation in 2D

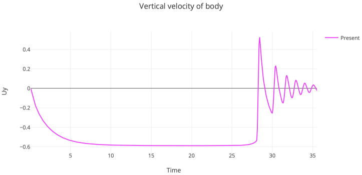




The differences in vertical velocity can be explained by a higher repulsion force intensity used for literature results.
[D. Wan, S. Turek, 2004]

Simulation in 3D

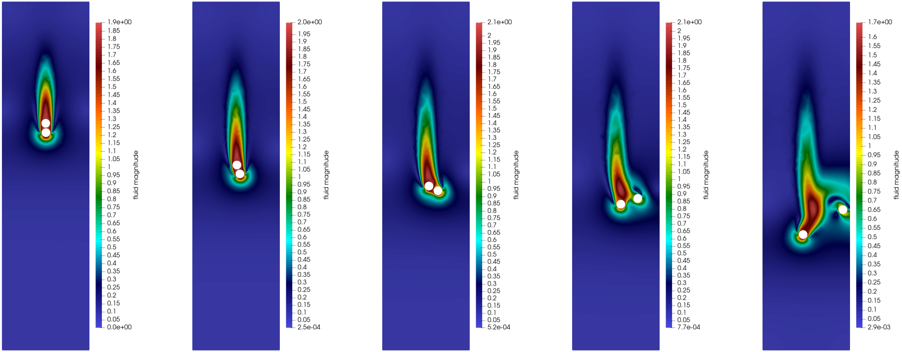




The sphere bounces at the bottom of the box before reaching a stationary position. This behavior during a sphere-bottom interaction is present in literature.
[S.M Dash, T.S Lee, 2015]

Simulation : Two disks interaction

Drafting, kissing and tumbling phenomenon in 2D

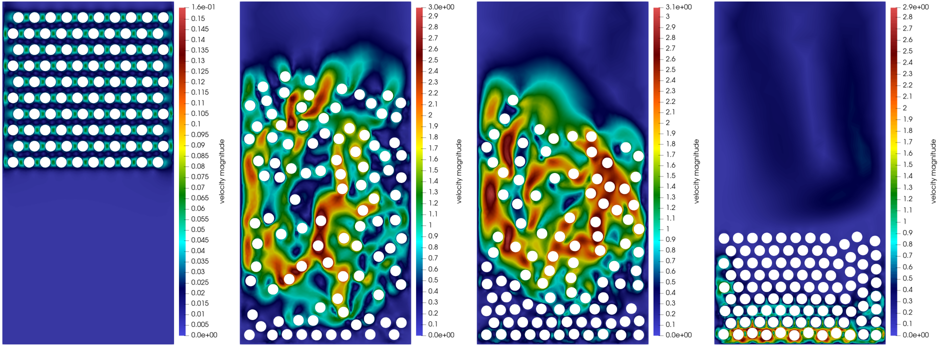


$t = 1.5$s $\quad$ $t = 2.0$s $\quad$ $t = 2.25$s $\quad$ $t = 2.5$s $\quad$ $t = 3.0$s

K. Usman, K. Walayat, R. Mahmood, et al. *Analysis of solid particles falling down and interacting in a channel with sedimentation using fictitious boundary method*, 2018.

Decheng Wan, Stefan Turek. *Direct Numerical Simulation of Particulate Flow via Multigrid FEM Techniques and the Fictitious Boundary Method*, 2004.

Simulation : 100 disks interaction in 2D



$t = 0.0$s | $t = 1.5$s | $t = 2.0$s | $t = 3.5$s

L.H.Juarez, R.Glowinski, T.W.Pan. *Numerical simulation of fluid flow with moving and free boundaries*, 2004.

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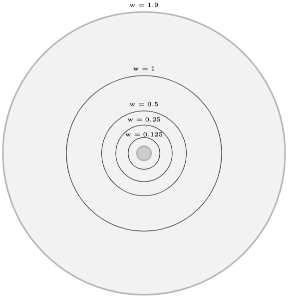
Tool : Narrow band fast marching method

- FMM computes distance function $D_i(X) : \mathbb{R}^d \longrightarrow \mathbb{R}$ starting from a front.
- Difficulty : execution costs.
- Solution : narrow band approach. $D_i(X)$ computed in near neighborhood around front.



Bandwidth	Execution time in 2D	Speed-up
1.9	1.956 s	/
1.0	0.479 s	4.08
0.5	0.143 s	3.35
0.25	0.052 s	2.75
0.125	0.026 s	2.00
Bandwidth	Execution time in 3D	Speed-up
1.9	22.854 s	/
1.0	3.930 s	5.82
0.5	0.686 s	5.73
0.25	0.184 s	3.73
0.125	0.093 s	1.98

Performance of narrow band approach

Narrow band fast marching method implemented by Thibaut METIVET.

Collisions with complex shaped rigid bodies

Reformulation of $\overrightarrow{F_{ij}}$ for body-body and $\overrightarrow{F_i^W}$ for body-wall interaction :

$$|\overrightarrow{F_{ij}}| = \begin{cases} 0\ , & \text{for} \quad d_{ij} > \rho, \\ \frac{1}{\epsilon}(\arg\min_{X \in \partial B_i} D_j(X) - \arg\min_{X \in \partial B_j} D_i(X))(\rho - d_{ij})^2\ , & \text{for} \quad 0 \leq d_{ij} \leq \rho. \end{cases}$$

$$|\overrightarrow{F_i^W}| = \begin{cases} 0\ , & \text{for} \quad d_i > \rho, \\ \frac{1}{\epsilon_W}(\arg\min_{X \in \partial B_i} D_\Omega(X) - \arg\min_{X \in \partial \Omega} D_i(X))(\rho - d_i)^2\ , & \text{for} \quad 0 \leq d_i \leq \rho. \end{cases}$$

Angular momentum of $\overrightarrow{F_i} = \overrightarrow{F_{ij}} + \overrightarrow{F_i^W}$ for body-body and body-wall interaction :

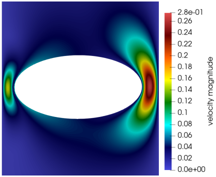
$$\frac{d[RJ_iR^T\omega_i]}{dt} = T_i - \overrightarrow{Gx_r} \times \overrightarrow{F_i},$$

where G the mass center of the body and x_r the point where $\overrightarrow{F_i}$ applies on the body B_i.

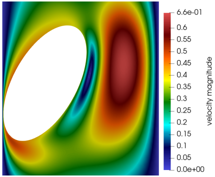
Tsorng-Whay Pan, Roland Glowinski, Giovanni P.Galdi. *Direct simulation of the motion of a settling ellipsoid in Newtonian fluid*, 2001.

Algorithm

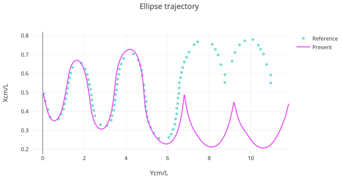
1. **Definition of collision parameters :** Read json file and stock parameters.
2. **Mesh preprocessing :** Associate boundary markers to the bodies and fluid domain.
3. **Contact detection :** Compute $D_i(X)$ to each boundary marker, evaluate $\arg\min_{X \in \partial B_j} D_i(X)$ for all bodies B_j, $j \neq i$, and stock data (coordinates of contact points, coordinates of mass centers and distance between bodies) in map if collision is taken place, i.e. $\arg\min_{X \in \partial B_j} D_i(X) \leq \rho$.
4. **Computation of collision force and torque :** Read map and use data to compute collision force and torque.
5. **Add collision force and torque :** Add collision force and torque to Newton's equations.



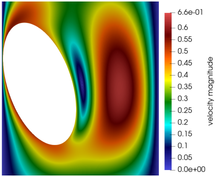
$t = 1.95$s



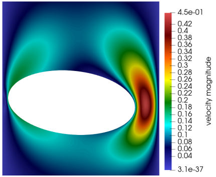
$t = 2.50$s



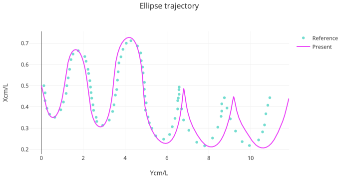
Validation with reference results



$t = 2.80$s



$t = 3.10$s

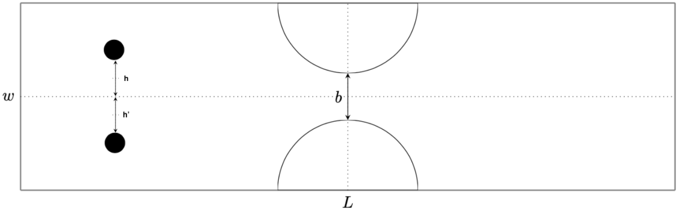


Validation with symmetrical results for rotations

The ellipse oscillates between the walls until it reaches a horizontal position at $t = 1.95$s. Then, it rotates around a single wall.

[Z. Xia, K. W. Connington, S. Rapaka, P. Yue, J. J. Feng, S. Chen, 2008]

Simulation : Particles suspension in a 2D symmetric stenotic artery



Parameter	Value
Particle diameter	d
L	$32d$
w	$8d$
b	$1.75d$
h	$2d + \frac{d}{4000}$
h'	$2d$
Mesh elements	103496

H. Li, H. Fang, Z. Lin, S. Xu, S.Chen. *Lattice Boltzmann simulation on particle suspensions in a two-dimensional symmetric stenotic artery*, 2004.

J. Wu, C. Shu. *Particulate Flow Simulation via a Boundary Condition-Enforced Immersed Boundary-Lattice Boltzmann Scheme*, 2009.

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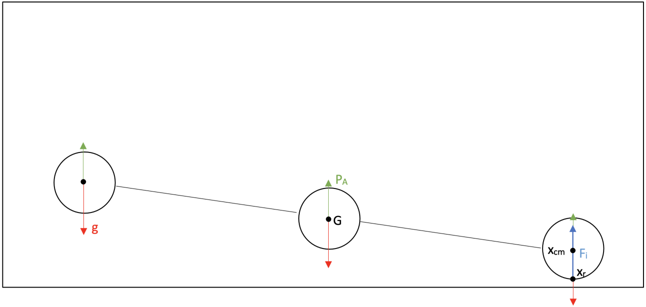
Collisions with articulated rigid bodies



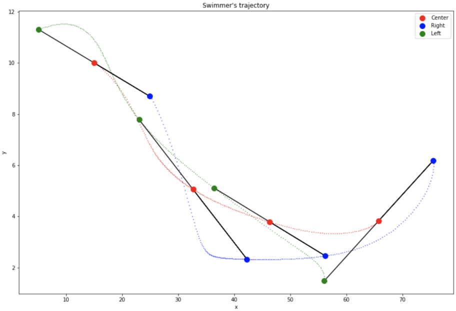
Newton's equation describing the angular velocity :

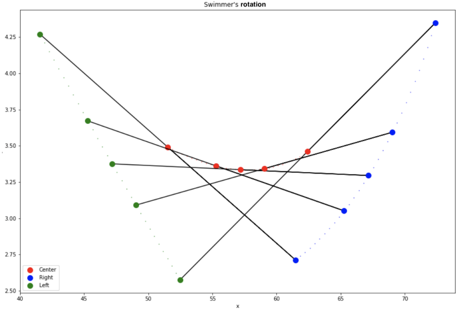
$$\frac{d[RJ_iR^T\omega_i]}{dt} = T_i - \overrightarrow{Gx_r} \times \overrightarrow{F_i} + \overrightarrow{Gx_{cm}} \times \overrightarrow{(g - P_A)},$$

where G the mass center of the swimmer, x_r the point where $\overrightarrow{F_i}$ applies on the body, x_{cm} the mass center of one body, g the gravity vector and P_A the buoyancy.

A. Najafi, R. Golestanian. *A Simplest Swimmer at Low Reynolds Number : Three Linked Spheres*, 2018.

Simulation : Mobile three-sphere swimmer near boundary in 2D





Swimmer's motion :

- Swimmer approaches the bottom of the channel.
- Right sphere arrives in lubrication zone : swimmer begins to change direction.
- Collision forces force the swimmer to move upwards.

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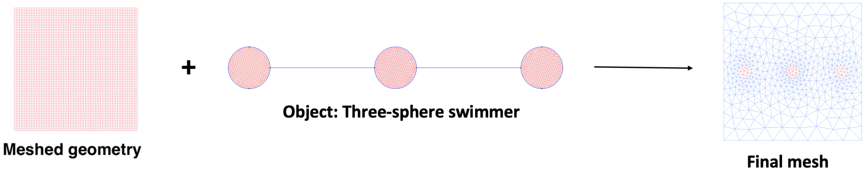
Application : Creation of objects inside a meshed geometry

Objective : Inserting rigid objects of complex shape in meshed fluid domains, in order to simulate the fluid-body interaction.

Algorithm :

1. Rotate, scale and translate mesh nodes to localize object in desired position in meshed geometry.
2. Build level-set function of object using Fast Marching method.
3. Interpolate the level-set function on meshed geometry.
4. Remesh using MMG functionality.

First example :



www.mmgtools.org.

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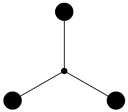
Other results

Simulations and benchmarks

- Validation and verification of algorithms.
- Influence of number of interacting bodies on execution time.
- Simulations : falling concave body, falling object in complex domain, etc.

Other internship results

- Resolution of Stokes equations.
- Analysis of motion of the three-sphere planar swimmer.
- Analysis of other collision models.



Conclusion and perspectives

Short-term perspectives

- Writing a paper on collision models.
- Further analyzing behavior of articulated bodies near wall.

Long-term perspectives

- Modeling and controlling the motion of deformable swimmers.
- Determining best swimming strategy.
- Objectives of my PhD project and of ANR project NEMO.

Conclusion

- Simulation of collisions between arbitrary bodies.
- Results in good agreement with literature.
- Efficiency of algorithms.

Thank you for your attention !

Do you have any questions ?

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